PHYS 301
Electricity and Magnetism

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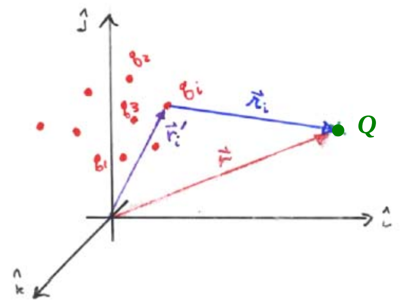
Today!

- Quiz!!
- Intro to cylindrical and spherical coordinates
- The Gradient
- The Divergence
- The Curl

Position & Displacement Vectors

- Coulomb's Law: $\vec{F} = \frac{1}{4\pi\varepsilon_o}\sum_{i=1}^{n}\frac{q_i\,Q}{\mathscr{r}_i^{\,2}}\hat{\mathscr{r}}_i$
- Position vectors: $\vec{r}_i$
- Displacement vectors: $\vec{\mathscr{r}}_i = \vec{r} - \vec{r}_i{}'$



$\vec{\mathscr{r}}_i \equiv \vec{r} - \vec{r}_i{}'$

CYLINDRICAL COORDINATES

$r = s =$ POLAR COORDINATE – RADIAL (XY-PLANE)

$\phi =$ POLAR COORDINATE – ANGLE (X-Y-PLANE)

$z =$ CARTESIAN Z-COORDINATE

UNIT VECTORS: $\hat{r} = \hat{s},\ \hat{\phi},\ \hat{k}$

$x = r\cos\phi$

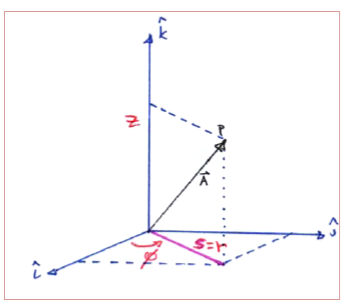
$y = r\sin\phi$

$z = z$

$\hat{r} = \hat{s} = \cos\phi\,\hat{i} + \sin\phi\,\hat{j}$

$\hat{\phi} = -\sin\phi\,\hat{i} + \cos\phi\,\hat{j}$

$\hat{k} = \hat{k}$



$s = \sqrt{x^2 + y^2}$

$\phi = \tan^{-1}[y/x]$

$z = z$

$\hat{i} = \cos\phi\,\hat{r} - \sin\phi\,\hat{\phi}$

$\hat{j} = \sin\phi\,\hat{r} + \cos\phi\,\hat{\phi}$

$\hat{k} = \hat{k}$

$d\vec{l} = dr\,\hat{r} + r\,d\phi\,\hat{\phi} + dz\,\hat{k}$

$d\tau = dV = r\,dr\,d\phi\,dz$

$d\vec{A}$ DEPENDS ON SURFACE!

DIV, GRAD, and CURL are no longer so simple!!

Differential Calculus

- For a function of one variable, *f(x)*,

  df/dx is the **slope** of the curve *f(x)*

- For a scalar function of two, three, or more variables [*e.g.*, g(x,y), h(x,y,z), *etc*.], the "**slope**" (how fast the function varies) depends upon the **direction** one moves
- The **GRADIENT** of the function serves as the generalization of the 1D derivative:

$$\vec{\nabla} P(x,y,z) \equiv \hat{i}\frac{\partial P}{\partial x} + \hat{j}\frac{\partial P}{\partial y} + \hat{k}\frac{\partial P}{\partial z}$$

in cartesian coords only!

Differential Calculus: The Gradient

- The del operator is defined as

$$\vec{\nabla} \equiv \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

Know this!

- Del acting on a scalar is called the **gradient**

$$\vec{\nabla} P(x,y,z) \equiv \hat{i}\frac{\partial P}{\partial x} + \hat{j}\frac{\partial P}{\partial y} + \hat{k}\frac{\partial P}{\partial z}$$